sophistication needed to provide for flexibility and safety, and as much simplicity as possible to insure dependability. Above all else, it must serve a useful purpose by providing to the user community data in a form and at a time so as to insure maximum utility in supporting its design functions.

Thrust Termination Analysis Utilizing an Aluminized Solid-Propellant Rocket Fuel

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Nomenclature

a,b	= burning rate constants
A	= area
C_W,C_d,C_F	= mass flow, discharge, and thrust coefficients
F	= thrust
g	= gravitational constant
N	= number of ports
P	= pressure
R	= gas constant
T	= temperature
TRR	= thrust reversal ratio
t	= time
V	= volume
$W,\dot{W}$	= flow and flow rate
α,β	= motor inner and outer stack angles to longitudinal axis of motor
γ	= ratio of specific heats
ρ	= density
ϵ	= expansion ratio
τ	= burning rate

Subscripts

A	= reverse thrust parallel to port centerline
AR	= reverse thrust parallel to motor centerline
a,c,e	= ambient, chamber, and exit conditions, respectively
b,g	= burning surface and gas, respectively
i	= condition along stack wall
M,P	= motor and propellant, respectively
R	= reversal port
S	= scarfed portion of port
SR	= scarfed portion of thrust parallel to main motor centerline
t	= main nozzle throat
vac	= vacuum condition

Introduction

ANALYSIS of the forces acting at the time of thrust termination requires the evaluation of all the vehicle forces. However, the parameter of greatest concern is the discharge coefficient. During the past several years, data have been collected by many investigators testing orifices of various inlet designs with air and/or nitrogen as a working fluid. This Note correlates those data with thrust termination data derived from a highly aluminized, double-base solid propellant exhausting through a straight-edge orifice. These hot-gas data were obtained from both static tests at ambient conditions and flight tests at near vacuum conditions. In addition, a simple one-dimensional analysis has been derived and utilized to predict the thrust reversal ratio under vacuum conditions.

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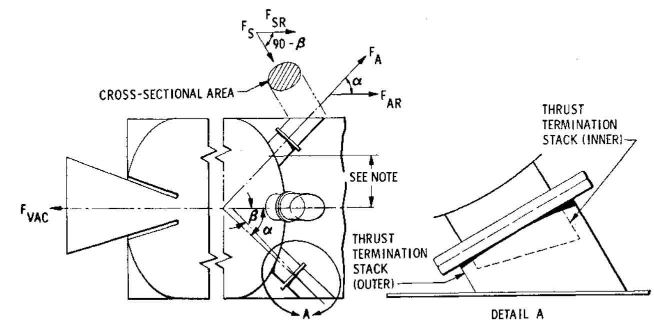

NOTE: ANGLE OF PORT ℄ AND MISSILE ℄ AND DISTANCE OF PORT CENTER TO MISSILE ℄ VARIES WITH CHAMBER PRESSURE

Fig. 1 Thrust termination schematic.

Figure 1 shows a schematic of the thrust termination system utilized on the C3 Poseidon. This system is initiated by a detonator that ignites a flexible linear-shaped charge and results in cleanly cut straight-edged venting orifices. For each port, there is a canted outer stack as shown in Fig. 1.

Mathematical Model

The general equations used to obtain the motor thrust prior to thrust termination can be derived from Zucrow[1] by assuming one-dimensional flow and are as follows:

$$F_{\mathrm{vac}} = P_C A_t C_{F\mathrm{vac}} \tag{1}$$

where

$$C_{F\mathrm{vac}} = C_d\{2\gamma^2/(\gamma - 1)[2/(\gamma + 1)]^{(\gamma+1)/(\gamma-1)} \times [1 - (P_e/P_c)^{(\gamma-1)/\gamma}]\}^{1/2} + (P_e/P_c)\epsilon \tag{2}$$

Using the terms in Fig. 1, the thrust contribution from the inner stack parallel to motor centerline can be expressed as

$$F_{AR} = P_c A_R C_{F\mathrm{vac}} \cos\alpha \tag{3}$$

and the force generated by the scarfed portion of the outer stack parallel to motor centerline can be given by

$$F_{SR} = P_S A_S \sin(90 - \beta) \tag{4}$$

Thus, the total thrust reversal is

$$F_R = N(F_{AR} - F_{SR}) \tag{5}$$

and the thrust reversal ratio is

$$TRR = F_R/F_{\mathrm{vac}} \tag{6}$$

The discharge coefficient (C_d) for the main nozzle is assumed to be near unity because of a well-designed entrance section; however, for the thrust termination port (a straight-edge orifice), this is not true. A theoretical C_d derivation from the mass-flow equation is

$$\dot{W}_{\mathrm{out}} = (C_{wt}C_d P_c A_R)_{\mathrm{ports}} + (C_{wt}P_c A_t)_{\mathrm{main\ nozzle}} \tag{7}$$

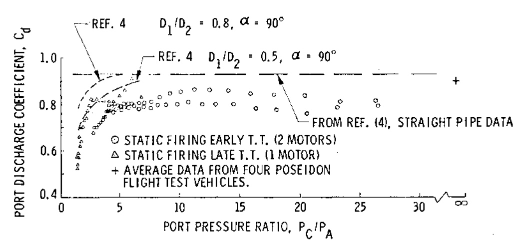

Fig. 2 Discharge coefficient vs pressure ratio.

and the rate at which gas is produced is

$$\dot{W}_g = A_b\rho_p\tau, \ \tau = P_c/(a + bP_c^{2/3}) \qquad (8)$$

From the conservation of mass,

$$dW/dt = A_b\rho_p\tau - P_cC_{wt}(C_dA_R + A_t) \qquad (9)$$

Since the major portion of gas venting occurs in less than 0.1 sec, the change in free volume is omitted. It is also assumed that the thermodynamic process is isothermal and that the perfect gas law is obeyed. Thus,

$$dP_c/dW = RT_c/V_c \qquad (10)$$

Inserting Eq. (10) into Eq. (9) and solving for C_d yields

$$C_d = (1/A_RP_cC_{wt})[A_b\rho_p\tau - (dP_c/dt)(V_c/RT)] - A_t/A_R \qquad (11)$$

Discussion

Kalt[2] evaluated the thrust-termination pressure decay of a solid propellant that was assumed to obey Vieille's law. Smoot and Isaacson[3] predicted $P_c(t)$ during the termination of solid-propellant motors using the Summerfield's burning-rate equation. In both cases, C_d was assumed to be constant. A constant C_d can be used in the thrust-termination analysis for highly aluminized gas products exhausting into near vacuum conditions; however, C_d does not remain constant for static motors fired at ambient conditions. Analysis at ambient conditions has to be accomplished on a given design because C_d is affected by the port-pressure ratio (P_c/P_a) and inlet geometry. Reference 4 reports experiments using different conical jet nozzles consisting of outlet-inlet diameter ratios from 0.50–0.91, nozzle-pressure ratios from 1.0–2.8, and nozzle half-angles of 5 to 90°. The data[4] for outlet-inlet ratios of 0.5 and 0.8, and for a half angle of 90° are shown in Fig. 2. Static-firing data from Hercules, Inc. and the data from Poseidon flight-test vehicles for a large range of P_c/P_A also are shown in Fig. 2. In comparison, the static-test data show a reasonable correlation to the data of Ref. 4. The discharge coefficients shown in Fig. 2 were calculated from Eq. (11) using Summerfield's burning-rate law. In the analysis of Fig. 2, it can be seen that as P_c/P_A increases (decreasing back pressure), C_d will increase and approach a maximum value. From Poseidon flight-test data, values of this coefficient were calculated and found to be approximately 0.90. Other investigators[5–7] have conducted similar experiments and have presented their data in terms of Reynolds number rather than P_c/P_A. The results of these data show the same trend as presented in Fig. 2.

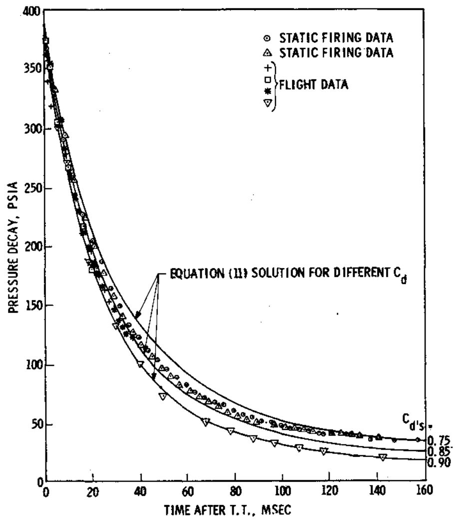

Fig. 3 Pressure decay curves during thrust termination.

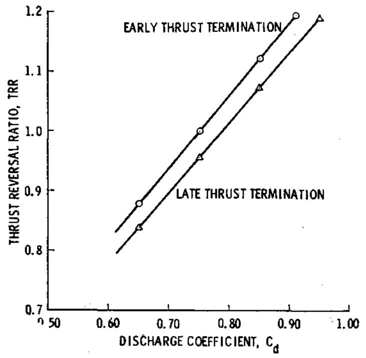

Fig. 4 Predicted thrust reversal ratio vs discharge coefficient.

Figure 3 presents data taken during the thrust-termination decay from static and flight tests. Equation (11) [for dPc/dt] was solved for different orifice coefficients, and these solutions are also shown in Fig. 3. For failure analysis, this type of presentation is useful in determining if all ports have been cut by the linear shaped charge and successfully ejected.

Data from static-test instruments placed in the inner and outer stacks provided the necessary pressure distribution along the stack walls to obtain solutions for Eqs. (3) and (4). The pressure ratio P_i/P_c near the exit plane of the inner stack was determined as 0.30. The pressure distribution along the scarfed portion of the outer stack was constant (P_i/P_c = 0.20), and this value was used in determining the solution of Eq. (4). In calculating Eq. (3), the flow through the inner stack was assumed to expand one-dimensionally and isentropically to an expansion ratio $\epsilon = 1.31$ and to a pressure ratio $P_i/P_c = 0.30$. No expansion along the outer stack was assumed.

For the final design, thrust reversal ratios have been calculated under vacuum conditions for a number of discharge coefficients; and typical curves for estimating performance for early and late thrust termination are provided in Fig. 4. The difference in these curves is due to main-nozzle-erosion characteristics. Once data are obtained from a flight, Eq. (11) is solved for C_d, and the TRR is determined from Fig. 4.

If TRR data are desired at lower altitudes where the main nozzle and thrust-termination ports are exhausting into an atmosphere consisting of back pressure (P_c/P_a), Eq. (2) will have to be modified to include $(P_e - P_a)\epsilon/P_c$ instead of $(P_e/P_c)\epsilon$; and the solution of Eqs. (1–6 and 11) can be used to yield TRR values for other than vacuum conditions. Note in this case that it is important to calculate the proper value of C_d for use in Eq. (3).

References

[1] Zucrow, M. J., *Aircraft & Missile Propulsion*, 1st ed., Wiley, New York, 1958, Chap. 10.

[2] Kalt, S., "Thrust Termination in Solid Rocket Motors—Evaluation of Ballistic Test Data," *ARS Journal*, Vol. 31, No. 1, Jan. 1961, pp. 84–86.

[3] Smoot, L. D. and Isaacson, L. K., "Prediction of Chamber Pressure Decay Transients During Termination of Solid Propellant Rocket Motors," *AIAA Journal*, Vol. 1, No. 8, Aug. 1963, pp. 1934–1935.

[4] Grey, R. E. and Wilsted, H. D., "Performance of Conical Jet Nozzles in Terms of Flow and Velocity Coefficients," TR 933, 1949, NACA.

[5] Carley, C. T., Jr. and Smetana, F. O., "Experiments on Transition Regime Flow Through a Short Tube With a Bellmouth Entry," *AIAA Journal*, Vol. 4, No. 1, Jan. 1966, pp. 47–54.

[6] Marchman, J. F., III, "Entrance Configuration Effects on Tube Flow in the Transition Regime," *AIAA Journal*, Vol. 7, No. 11, Nov. 1969, pp. 2147–2148.

[7] Sreekanth, A. K., "An Experimental Investigation of Mass Flow Through Short Circular Tubes in the Transition Flow Regime," Rept. DI-82-0427, April 1965, Boeing Scientific Research Lab.